

# Improving CRO performance using stepwise approach for constrained optimisation

S L Pandharipande, A K Dixit

**Abstract**—Chemical reaction optimisation (CRO) is amongst newer methods of evolutionary algorithms that are being developed suited in searching global solutions to varied nature of optimisation problems. It is, nature inspired meta-heuristics method and chemical reaction mechanism is the source of inspiration behind its. In a chemical reaction, the reactants; the unstable substances; are converted into products; the relatively stable ones. The reactants with some initial energy interact with each other through a sequence of elementary steps. At the end, molecules with minimum energy to support their stable structure are formed. Based on the present work, the results of the numerical experimentation, it is concluded that the values obtained from the stepwise approach are way better than those obtained using CRO without stepwise search and using conventional techniques such as geometric programming.

**Index Terms**—Chemical reaction optimisation, non-linear constraints, non-linear optimisation problems, MATLAB, stepwise approach, search interval selection

## 1 INTRODUCTION

Several processes and operations are involved in various spheres of activities. The outcome of these processes is usually dependent upon the parameters or variables involved. The efficient processes are those operating at the optimal conditions defined by proper combination of these variables. Due to a very large number of dependent and independent parameters involved in many industrial processes, the decision making to run them optimally becomes complex, tedious and time consuming, sometimes even inaccurate and erratic (It is usually based on the correlations for these variables and thereafter searching optimal solutions). The present scenario related to developing correlations and searching optimal solutions however is mixed. Due to advent of fast computational techniques newer methods such as evolutionary algorithms are being viewed as an alternative to solve these modelling and optimisation situations.

'Chemical Reaction Optimisation' is one such evolutionary method of searching optimal solution with main advantage of universal applicability. There are disadvantages that are associated with it, that are commonly observed in any random search method such as lack of definitive direction in search. The objective of the present work is to make CRO more definitive in search of optimal solution.

Present work aims at improvement of performance of CRO using MATLAB®. Test runs are conducted with constant CRO parameters to obtain the best optimal values. The best five consecutive runs are tabulated with the corresponding values of variables to select the search interval for the next step. Numbers of steps are continued till no further significant improvements in the function values are obtained.

## 2 NUMERICAL EXPERIMENTS

Two types of test objective functions involving four and three

variables have been considered for numerical experimentation and the comparison is carried out among the best optimal values obtained using these approaches.

### 2.1 Methodology

The methodology adapted is as given below:

1. The CRO parameters like appropriate popsize, lossrate and number of iterations are selected using few trial and error runs in step 1.
2. Five initial test runs are conducted in step 2 using entire search interval using the CRO parameters as decided in step 1.
3. In step 3, the decision regarding the selection of search interval is taken for conducting further runs based on the function values obtained in step 2.
4. The process followed in step 3 is repeated for conducting further runs, thereby narrowing the search interval.
5. The criteria of termination is based on the close proximity of the function values and longer run times

### 2.2 Test Function 1

Non-Linear Objective function with non-linear inequality constraints:

Objective function involving 4 variables is selected as given below.

Function:

Minimize

$$F(x_1, x_2, x_3, x_4) = 20 * x_2 * x_3 * x_4^4 + 20 * x_1^2 * x_3^{-1} + 5 * x_2 * x_3^4$$

Subject to:

$$5 * x2^{-5} * x3^{-1} \leq 1$$

$$10 * x1^{-1} * x2^3 * x4^{*1} \leq 1$$

$$xi > 0 (i = 1,2,3,4)$$

Conventional Technique: The objective function has been solved as reported in the literature, using geometric programming to obtain minimum values.

Chemical Reaction Optimisation (CRO) Technique: The present work optimizes the function by employing the developed CRO algorithm, with and without stepwise approaches.

The details of appropriate pop-size, ke lossrate and iterations selected based on trial and error runs are as given in table 1.

Based on values of the test function 1 obtained, decision regarding the interval selection of the 4 variables for the next five runs is taken. The details of the function values and their corresponding variable values are given in table 3.

The process is repeated for three more times. The details are given in table 4.

The criteria for termination of further narrowing of search interval is based on the close proximity of the function values and longer run times. Similarly 25 runs are carried out without stepwise approach keeping Ke lossrate, pop-size and iterations constant as given in table 1. The values of test functions obtained using with and without stepwise approaches for 25 runs are listed in table 5.

The optimal value of the test function 1 and the corresponding values of variables are reported in literature that are obtained using geometric programming. The details are given in the table 6.

**TABLE I**  
Constant values of Various Parameters Selected

Popsize	Kelossrate	Iterations
4	.001	500

**TABLE II**  
Details of Five Initial Test Runs for Function 1

Sr. No.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
1	0	10	0	10	0	10	0	10	<b>15722</b>	7.3525	1.3704	1.3584	4.4737
2	0	10	0	10	0	10	0	10	<b>3672</b>	9.7752	1.2245	7.4161	2.0289
3	0	10	0	10	0	10	0	10	<b>5377</b>	5.5287	1.0237	7.7103	2.3716
4	0	10	0	10	0	10	0	10	<b>1916.9</b>	9.2839	0.9268	8.8096	1.6993
5	0	10	0	10	0	10	0	10	<b>4161.5</b>	8.8606	0.9057	9.8888	2.1114

**TABLE III**  
Details of interval selection for next 5 runs

Sr. No.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
1	5	10	0	2	0	10	0	5	<b>806.50</b>	9.7359	0.9918	7.4494	1.1699
2	5	10	0	2	0	10	0	5	<b>810.78</b>	9.4151	0.9667	6.8358	1.252
3	5	10	0	2	0	10	0	5	<b>1010.9</b>	6.0474	0.9108	9.3756	1.3289
4	5	10	0	2	0	10	0	5	<b>968.72</b>	7.6986	1.0285	4.556	1.5919
5	5	10	0	2	0	10	0	5	<b>4643.2</b>	6.952	1.0646	9.0906	2.1449

**TABLE IV**  
Details of additional run for test function

S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
6	6.5	10	0	1	5	10	0	1.5	<b>747.63</b>	8.451	0.9062	8.6391	1.1175
7	6.5	10	0	1	5	10	0	1.5	<b>618.90</b>	8.9422	0.9712	6.5952	1.0656
8	6.5	10	0	1	5	10	0	1.5	<b>761.77</b>	9.9581	0.9562	6.2775	1.2101
9	6.5	10	0	1	5	10	0	1.5	<b>566.06</b>	8.7673	0.9679	6.0429	1.0363
10	6.5	10	0	1	5	10	0	1.5	<b>628.13</b>	9.3929	0.9654	6.8353	1.0228
S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
11	8	9.5	0	1	6	8.5	0	1	<b>589.43</b>	9.2962	0.9588	6.4616	0.9957
12	8	9.5	0	1	6	8.5	0	1	<b>598.52</b>	8.8046	0.9412	7.5331	0.9702
13	8	9.5	0	1	6	8.5	0	1	<b>577.61</b>	9.0826	0.9302	7.6087	0.8967
14	8	9.5	0	1	6	8.5	0	1	<b>594.25</b>	9.2033	0.959	6.8152	0.9849
15	8	9.5	0	1	6	8.5	0	1	<b>592.3219</b>	8.9095	0.9002	8.4821	0.8542
S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
16	8.5	9.5	0	1	6.5	7.5	0	1	<b>572.1335</b>	8.9952	0.9555	6.5587	0.9889
17	8.5	9.5	0	1	6.5	7.5	0	1	<b>559.06</b>	8.7266	0.9411	6.888	0.9698
18	8.5	9.5	0	1	6.5	7.5	0	1	<b>554.2687</b>	8.2891	0.9314	7.1503	0.9822
19	8.5	9.5	0	1	6.5	7.5	0	1	<b>564.9919</b>	9.192	0.9236	7.4613	0.8767
20	8.5	9.5	0	1	6.5	7.5	0	1	<b>562.1468</b>	9.3386	.9231	7.4875	.8449
S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
6	6.5	10	0	1	5	10	0	1.5	<b>747.63</b>	8.451	0.9062	8.6391	1.1175
7	6.5	10	0	1	5	10	0	1.5	<b>618.90</b>	8.9422	0.9712	6.5952	1.0656
8	6.5	10	0	1	5	10	0	1.5	<b>761.77</b>	9.9581	0.9562	6.2775	1.2101
9	6.5	10	0	1	5	10	0	1.5	<b>566.06</b>	8.7673	0.9679	6.0429	1.0363
10	6.5	10	0	1	5	10	0	1.5	<b>628.13</b>	9.3929	0.9654	6.8353	1.0228
S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
11	8	9.5	0	1	6	8.5	0	1	<b>589.43</b>	9.2962	0.9588	6.4616	0.9957
12	8	9.5	0	1	6	8.5	0	1	<b>598.52</b>	8.8046	0.9412	7.5331	0.9702
13	8	9.5	0	1	6	8.5	0	1	<b>577.61</b>	9.0826	0.9302	7.6087	0.8967
14	8	9.5	0	1	6	8.5	0	1	<b>594.25</b>	9.2033	0.959	6.8152	0.9849
15	8	9.5	0	1	6	8.5	0	1	<b>592.3219</b>	8.9095	0.9002	8.4821	0.8542
S.N.	limits of x1		limits of x2		limits of x3		limits of x4		function	x1	x2	x3	x4
	lower	upper	lower	upper	lower	upper	lower	upper					
16	8.5	9.5	0	1	6.5	7.5	0	1	<b>572.1335</b>	8.9952	0.9555	6.5587	0.9889
17	8.5	9.5	0	1	6.5	7.5	0	1	<b>559.06</b>	8.7266	0.9411	6.888	0.9698
18	8.5	9.5	0	1	6.5	7.5	0	1	<b>554.2687</b>	8.2891	0.9314	7.1503	0.9822
19	8.5	9.5	0	1	6.5	7.5	0	1	<b>564.9919</b>	9.192	0.9236	7.4613	0.8767
20	8.5	9.5	0	1	6.5	7.5	0	1	<b>562.1468</b>	9.3386	.9231	7.4875	.8449

**TABLE V**  
 Function values for consecutive 25 runs with and without stepwise approaches

S. No.	Function Value	
	stepwise	without
1	15722	1837
2	3672	4438.7
3	5377	7400.5
4	1916.9	7683.3
5	4161.5	7691.6
6	806.50	4824.2
7	810.78	9025.5
8	1010.9	930.56
9	968.72	1969.6
10	4643.2	7239.2
11	747.63	2920.1
12	618.90	2801.7
13	761.77	9597.9
14	566.06	2373.1
15	628.13	4587.2
16	589.43	2178.4
17	598.52	781.29
18	577.61	5616.5
19	594.25	590.4
20	592.32	5927.4
21	572.13	1941.7
22	559.06	7111.3
23	554.26	748.4
24	564.99	687.57
25	562.14	919.76

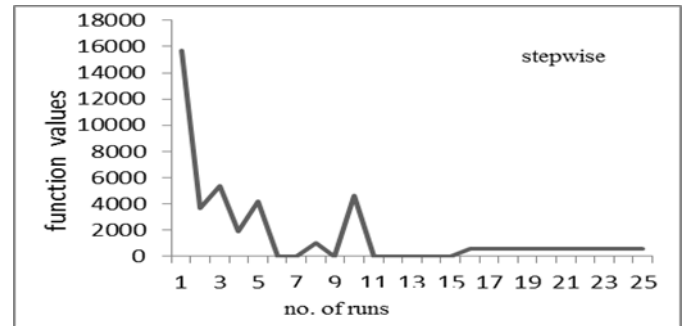


Fig. 2. Variation in test function 1 values with step wise approach

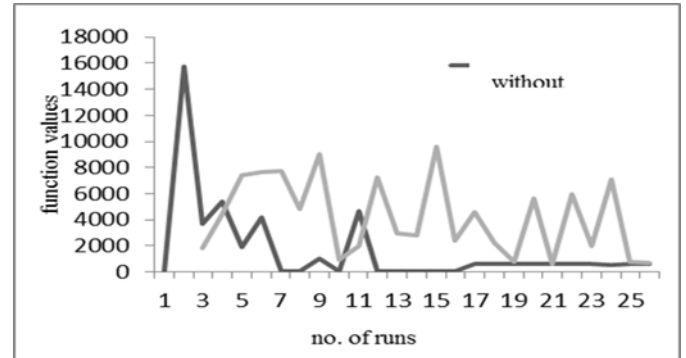


Fig. 3. Comparison between the test function 1 values obtained with and without stepwise approach

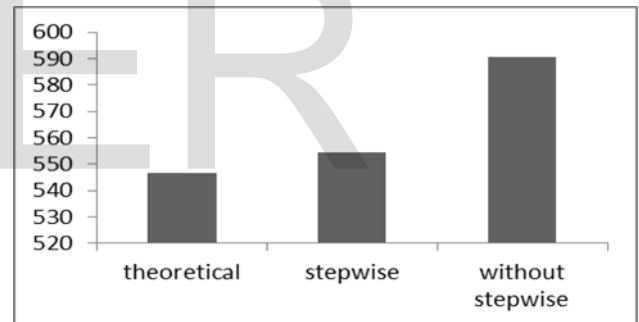


Fig. 4. Comparison between best values obtained

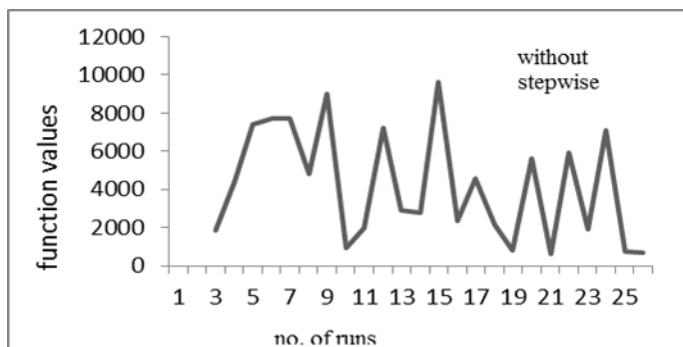


Fig. 1. Variation in test function1 values without step wise approach

**TABLE VI**

Value of the test function and the corresponding values of variables as reported in literature

Approach	Function value	X1 value	X2 value	X3 values	X4 values
Theoretical	<b>546.6402</b>	8.6365	.9397	6.8219	.9609
Stepwise	<b>554.2687</b>	8.2891	0.9314	7.1503	0.9822
Without stepwise	<b>591.8452</b>	9.2033	.959	6.8152	.98

**2.3 Test Function 2**

Non-Linear Objective function with non-linear inequality constraints:

Objective function involving 3 variables is selected as given below

Function: Minimize:  $F(x_1, x_2, x_3): 1/(x_1 \cdot x_2^2 \cdot x_3^2)$

Subject to:  $x_1^2 + x_2^2 + x_3 \leq 1$   
 $x_i > 0 (i = 1, 2, 3, 4)$

Conventional Technique: The objective function has been solved as reported in the literature, using geometric programming to obtain minimum values.

Chemical Reaction Optimisation (CRO) Technique: The present work optimizes the function by employing the developed CRO algorithm

Details of appropriate pop-size, ke lossrate and iterations selected using trial and error runs are as given in table 7.

The details of optimal values obtained in five initial test runs are as listed in the table 8.

Based on the values of the function obtained, decision regarding the interval selection of the 4 variables for the next

two runs is taken. The details of the function values and corresponding variables values are given in the table 9.

The criteria for termination of further narrowing of search interval is based on close proximity in the function values and longer run times for consecutive runs. Similarly 15 runs are carried out without stepwise approach keeping Ke lossrate, pop-size and iterations constant as given in table 10. The value of the test function and the corresponding values of variables are reported in literatures that are obtained using geometric programming. The details are given in the table 11.

TABLE VII  
 Constant values of various parameters

Ke lossrate	Pop-size	Iterations
.000001	4	100

TABLE VIII  
 Details of five initial test runs for test function 2

Sr. No.	limits of x1		limits of x2		limits of x3		function	x1	x2	x3
	lower	upper	lower	upper	lower	upper				
1	0	10	0	10	0	10	56.1499	0.4833	0.739	0.2598
2	0	10	0	10	0	10	197.3527	0.391	0.1464	0.7777
3	0	10	0	10	0	10	44.6367	0.3433	0.4019	0.6357
4	0	10	0	10	0	10	68.0619	0.3579	0.3527	0.5754
5	0	10	0	10	0	10	49.7729	0.3853	0.3474	0.6573

TABLE IX  
 Details of interval selection for next 3 runs

Sr. No.	limits of x1 selected		limits of x2 selected		limits of x3 selected		function	x1	x2	x3
	lower	upper	lower	upper	lower	upper				
6	0	0.5	0	0.8	0	0.8	30.9983	0.4442	0.4444	0.6064
7	0	0.5	0	0.8	0	0.8	29.8267	0.3537	0.5594	0.5504
8	0	0.5	0	0.8	0	0.8	31.2878	0.4268	0.6062	0.4514
9	0	0.5	0	0.8	0	0.8	50.047	0.4226	0.3062	0.7101
10	0	0.5	0	0.8	0	0.8	30.3483	0.4113	0.4333	0.6533
Sr. No.	limits of x1		limits of x2		limits of x3		function	x1	x2	x3
	lower	upper	lower	upper	lower	upper				
11	0	0.5	0	0.6	0	0.6	25.9295	0.4699	0.5973	0.4796
12	0	0.5	0	0.6	0	0.6	35.5818	0.3972	0.451	0.5899
13	0	0.5	0	0.6	0	0.6	26.0642	0.4549	0.5216	0.5567
14	0	0.5	0	0.6	0	0.6	40.0062	0.3846	0.5517	0.4621
15	0	0.5	0	0.6	0	0.6	24.6093	0.4368	0.5648	0.54

TABLE X

Run no.	Function value	
	stepwise	Without stepwise
1	56.1499	142.4666
2	197.3527	39.546
3	44.6367	35.7804
4	68.0619	52.843
5	49.7729	71.426
6	30.9983	44.6367
7	29.8267	81.0587
8	31.2878	49.7729
9	50.047	122.9079
10	30.3483	34.5196
11	25.9295	28.4401
12	35.5818	62.1041
13	26.0642	146.6323
14	40.0062	29.4638
15	24.6093	33.7437

Test function 2 values for consecutive 15 runs with and without stepwise approaches

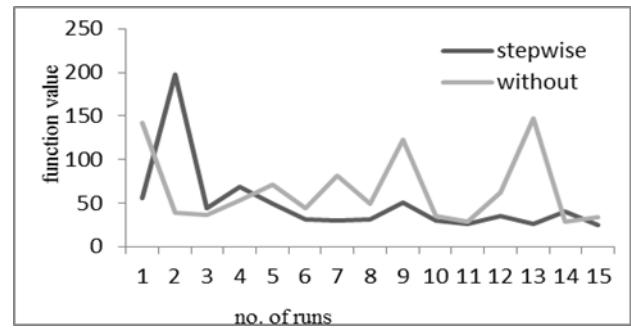


Fig. 7 Comparison between the test function 2 values obtained with and without stepwise approach

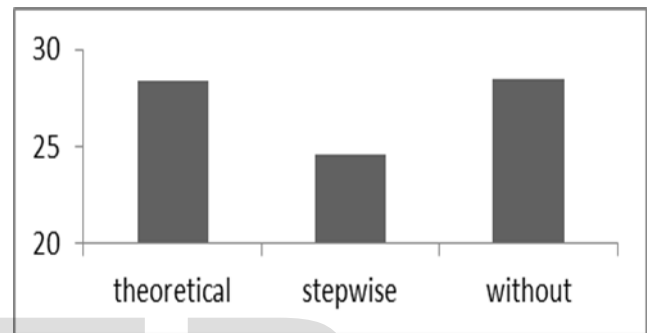


Fig. 8 Comparison between the best obtained value

TABLE XI

Value of the test function 2 and the corresponding value of variables as reported in literature

Approach	Function value	X1 value	X2 value	X3 value
Theoretical	28.3617	.3780	.5345	.5714
Stepwise	24.6093	.4368	.5648	.54
Without stepwise	29.4638	.378	0.5245	0.5713

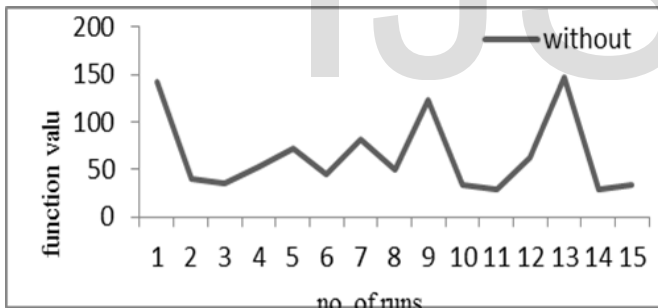


Fig. 5 Variation in test function 2 values without stepwise approach

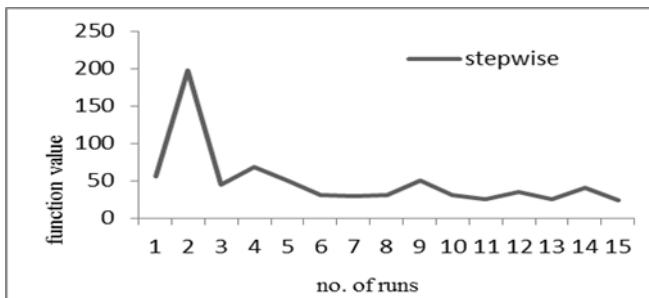


Fig. 6 Variation in test function 2 values with step wise approach

#### 4 RESULTS AND DISCUSSION

*Test Function 1:* Graphs are plotted to study the variation of function values obtained for various runs conducted without and with stepwise approach as shown in figure no. 1 and 2 respectively. As can be seen from these graphs the CRO algorithm has the natural behaviour of random search lacking direction, which is observed in this case as well.

The stepwise approach on the other hand has reduced the randomness in search towards optimal value and is more definitive in its direction to optimal value from step to step. This can be seen from the figure 3 depicting the comparison of function values obtained using with and without step. It substantiates the claim of up gradation of CRO algorithm by using stepwise approach that dampens the randomness from run to run and step to step making it more definitive.

Similarly figure 4 shows the comparison among the best

optimal values obtained using theoretical, stepwise and without stepwise approaches. As can be seen from this graphs, the performance of stepwise approach suggested in the present work is comparable with that of the literature value. With incorporating more steps and machine time it can be further improved

*Test Function 2:* For test function 2, the figures 5 and 6 show the graphs for function values obtained with and without stepwise approaches. As can be seen from these graphs the CRO algorithm without stepwise approach has shown the similar behaviour of lack of direction and randomness in the search as was observed for test function 1. However the search is more definitive and smoothed with step wise approach.

Figure 7 shows the graph, comparing the values obtained with and without stepwise approaches. As seen from the graph, the optimal function values obtained in stepwise approach is better as compared to values obtained without stepwise approach. Figure 8 shows the comparison between the best optimal value obtained using theoretical approach, with and without stepwise approach. The function values obtained using CRO is 13.23% better than literature value indicating better performance of stepwise approach suggested in the present work.

## 5 CONCLUSION

Non-linear constrained optimization involving more than two variables is of interest among researchers from different disciplines. Conventional methods have major limitations that they are problem specific and lack in universality of approach.

Newer methods like CRO are evolutionary algorithms and are making positive impact on addressing to these limitations. However as the basis of these algorithms is random search, they possess the lack of definitive direction in moving towards optimal solution.

Present work is aimed at improving existing CRO performance incorporating a feature of stepwise interval search selection approach. Numerical experimentation is conducted on two test functions 1 and 2 having four and three variables respectively with and without stepwise approaches.

Based on observation, results and discussion, it can be concluded that the performance of CRO is comparable with the conventional method, geometric programming in this case.

It can also be inferred that the improvement of CRO performance employing stepwise search interval selection approach has been successful. The newer approach is more definitive in its search interval selection in both the cases of the test functions studied in present work.

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